

Name: Key

Exercise 1. (5 points) Show the equation

$$3x^3 - 4x^2 + x - 1 = 0$$

has a solution between 1 and 2.

Let $f(x) = 3x^3 - 4x^2 + x - 1$. Since f is a polynomial, f is continuous on $(-\infty, \infty)$, specifically, it is continuous on $[1, 2]$. Then since $f(1) = 3 - 4 + 1 - 1 = -1$ & $f(2) = 3(8) - 4(4) + 2 - 1 = 9$, the Intermediate Value Theorem says that there exists a c in $(1, 2)$ such that $f(c) = 0$.

Exercise 2. (5 points) Find

$$\lim_{x \rightarrow 2^+} \frac{x}{2x-4} \quad \text{and} \quad \lim_{x \rightarrow 2^-} \frac{x}{2x-4}.$$

$$\lim_{x \rightarrow 2^+} \frac{x}{2x-4} = \infty \quad \text{since} \quad \lim_{x \rightarrow 2^+} x = 2 \quad \& \quad \lim_{x \rightarrow 2^+} \frac{1}{2x-4} = \infty$$

$$\lim_{x \rightarrow 2^-} \frac{x}{2x-4} = -\infty \quad \text{since} \quad \lim_{x \rightarrow 2^-} x = 2 \quad \& \quad \lim_{x \rightarrow 2^-} \frac{1}{2x-4} = -\infty$$

Note: We can think of $\lim_{x \rightarrow 2^+} \frac{x}{2x-4}$ as $\lim_{x \rightarrow 2^+} \left[(x) \left(\frac{1}{2x-4} \right) \right]$.