Test 2B MA125-6A October 2, 2013

Name: Key Signature: ____________

SHOW ALL YOUR WORK!

If you have time, find a way to check your answers.

Part 1

1. [6 points] Find y' if $y = x^2 \tan(x)$.

$$y' = \frac{d}{dx}(x^2)\tan(x) + x^2 \frac{d}{dx}(\tan(x))$$
$$= \boxed{2x\tan(x) + x^2 \sec^2(x)}$$

2. [6 points] Differentiate the function $v(z) = 5z^{40}$.

$$v'(z) = 5\frac{d}{dz}(z^{40})$$
$$= 5(40z^{39})$$
$$= \boxed{200z^{39}}$$

3. [6 points] Let $f(x) = u(v(x))$, where $v'(0) = -3$, $v(0) = 2$ and $u'(2) = 1$. Find $f'(0)$.

$$f'(x) = u'(v(x))v'(x)$$
$$f'(0) = u'(v(0))v'(0)$$
$$= u'(2)(-3)$$
$$= (1)(-3) = \boxed{-3}$$

4. [6 points] Find the derivative of the function $f(s) = -4s^3 + \dfrac{-5}{s^2} + 4s - \sqrt{7}$.

$$f'(s) = -4\frac{d}{ds}(s^3) - 5\frac{d}{ds}(s^{-2}) + 4\frac{d}{ds}(s) - \frac{d}{ds}(\sqrt{7})$$
$$= -4(3s^2) - 5(-2s^{-3}) + 4(1) - 0$$
$$= \boxed{-12s^2 + 10s^{-3} + 4}$$

5. [6 points] Find the derivative of the function $g(x) = (\sin(x))^{35}$.

$$g'(x) = 35(\sin(x))^{34}\frac{d}{dx}(\sin(x))$$

$$= \boxed{35.(\sin(x))^{34}\cos(x)}$$

6. [6 points] Find the values of x for which the curve $y = x^3 + 5x^2 - 8x + 2$ has a horizontal tangent line.

We want to find x so that $y'(x) = 0$.

$$y' = 3x^2 + 5(2x) - 8(1) + 0$$

$$= 3x^2 + 10x - 8$$

$$= 3x^2 - 2x + 12x - 8$$

$$= \cancel{3}x(3x-2) + 4(3x-2)$$

$$= (3x-2)(x+4)$$

So, $y'(x) = 0$ when

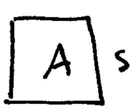
$3x - 2 = 0$ or $x + 4 = 0$

$3x = 2$ $\quad \boxed{x = -4}$

$\boxed{x = \frac{2}{3}}$

7. [6 points] Each side of the square is increasing at a rate of 4 cm/s. At what rate is the area of the square increasing when the area of the square is 9 cm^2?

Let s be the side length. Let A be the area.



We know: $\frac{ds}{dt} = 4\,\text{cm/s}$

We want: $\left.\frac{dA}{dt}\right|_{s=3\text{cm}}$

A & s are related by

$$A = s^2$$

So, $\frac{dA}{dt} = 2s\frac{ds}{dt}$

Thus,

$$\left.\frac{dA}{dt}\right|_{s=3} = 2(3)(4) = \boxed{24\ \text{cm}^2/\text{s}}$$

Part 2

1. [7 points] Find the equation of the tangent line to the parabola $y = x^2 - 8x + 9$ at the point (3,-6)

Since $y' = 2x - 8$, the slope of the tangent line when $x = 3$ is $y'(3) = 2(3) - 8 = -2$. Thus, the equation for the tangent line is

$$y + 6 = -2(x - 3)$$

$$y + 6 = -2x + 6$$

$$\boxed{y = -2x}$$

2. [10 points] Let $f(x) = x^2 - 5$. Use the **limit definition of the derivative** to find the derivative $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 5 - (x^2 - 5)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0} 2x + h = \boxed{2x}$$

3. [11 points] Use implicit differentiation to find the derivative $\frac{dy}{dx}$ if $y^2 = \sin(xy)$.

$$\frac{d}{dx}\left(y^2\right) = \frac{d}{dx}\left(\sin(xy)\right)$$

$$2y\frac{dy}{dx} = \cos(xy)\frac{d}{dx}(xy)$$

$$2y\frac{dy}{dx} = \cos(xy)\left(y + x\frac{dy}{dx}\right)$$

$$2y\frac{dy}{dx} = y\cos(xy) + x\cos(xy)\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$$

$$\frac{dy}{dx}\left(2y - x\cos(xy)\right) = y\cos(xy)$$

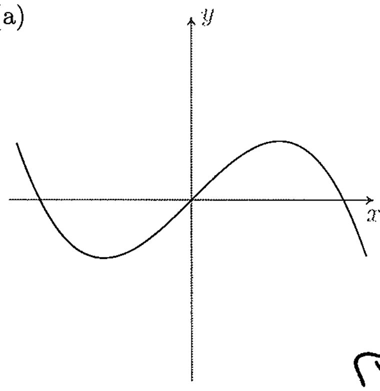
$$\boxed{\frac{dy}{dx} = \frac{y\cos(xy)}{2y - x\cos(xy)}}$$

4. [18 points] If $f(x) = \dfrac{x^2}{1+x}$, find $f''(1)$.

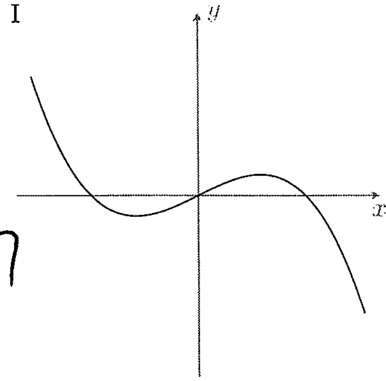
See Test 2A solution

5. [12 points] Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV.

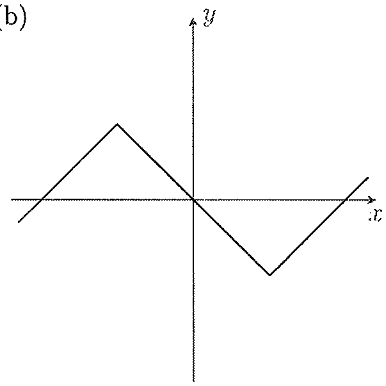
(a)



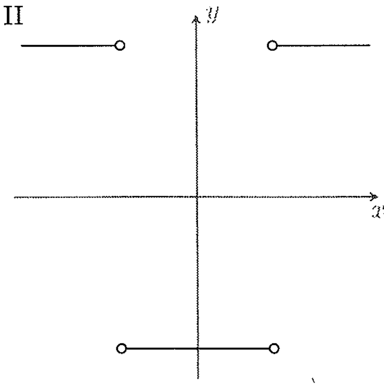
I



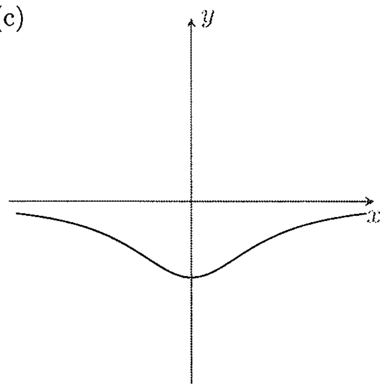
(b)



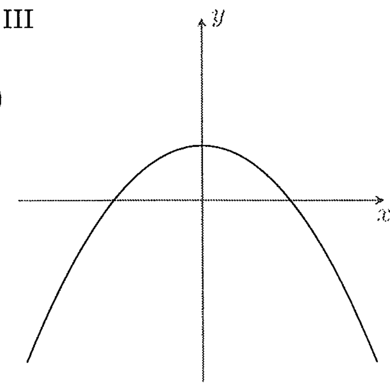
II



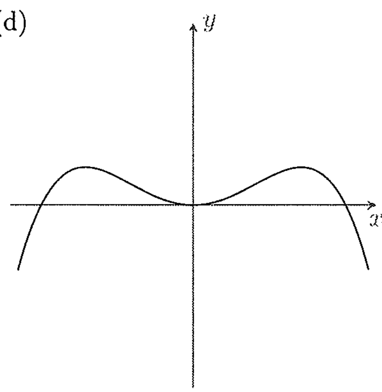
(c)



III



(d)



IV