Test 4 MA 125-6A November 20, 2013

Name: Key Signature: ____________

SHOW ALL YOUR WORK!

If you have time, find a way to check your answers.

Part 1

1. [5 points] Find all the critical points of $f(x) = x + \frac{3}{2}x^{2/3}$.

$$f'(x) = 1 + \frac{3}{2}\left(\frac{2}{3}x^{-1/3}\right) = 1 + \frac{1}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}+1}{\sqrt[3]{x}}$$

$f'(x)=0$ when $\sqrt[3]{x}+1=0 \Rightarrow \sqrt[3]{x}=-1 \Rightarrow x=-1$

$f'(x)$ DNE when $x=0$.

$\boxed{\text{Critical points are } x=0,\ x=-1}$

2. [5 points] Find the interval(s) in the x-axis where $y = f(x) = xe^x$ is **decreasing**.

$$f'(x) = xe^x + e^x = (x+1)e^x$$

Since $e^x > 0$ for all x, $f'(x) < 0$ when $x+1<0$. $x+1<0$ when $x<-1$. Thus, f is decreasing on $(-\infty, -1)$.

3. [5 points] Find the open interval(s) where the function $f(x) = x + \sin(x)$ is **concave up** on the interval $[0, 2\pi]$.

$f'(x) = 1 + \cos(x)$

$f''(x) = -\sin(x)$

Thus, $f''(x) > 0$ when $\sin(x) < 0$. On $[0, 2\pi]$, $\sin(x) < 0$ when $\pi < x < 2\pi$. Thus, f is concave up on $(\pi, 2\pi)$.

4. [5 points] Given the function $y = f(x) = x^4 - 5x$ find all the points of inflection (both the x and y coordinate of each point).

$f'(x) = 4x^3 - 5$

$f''(x) = 12x^2$

Since $f''(x) = 12x^2 \geq 0$ for all x, $f''(x)$ does not change sign. Thus, f has no inflection points.

5. [5 points] Find all the numbers c that satisfy the conclusion of Rolle's Theorem on the given interval.

$$h(x) = 3x^2 - 24x + 2 \text{ on } [2, 6]$$

Since h satisfies the conditions of Rolle's Theorem, we want to find c in $(2, 6)$ such that $h'(c) = 0$.

Since $h'(x) = 6x - 24$, we can solve

$$h'(x) = 0$$
$$6x - 24 = 0$$
$$6x = 24$$
$$\boxed{x = 4.}$$

6. [5 points] Find the most general form for the **antiderivative** F of

$$f(x) = x^2(2x + 12)$$

$$f(x) = x^2(2x+12) = 2x^3 + 12x^2$$

Then

$$F(x) = 2\left(\frac{x^4}{4}\right) + 12\left(\frac{x^3}{3}\right) + C$$
$$= \frac{1}{2}x^4 + 4x^3 + C.$$

Part 2

1. [15 points] Given the following function on the given interval

$$h(s) = s^3 - 3s + 2, \qquad [-2, 2]$$

verify that the function satisfies the hypotheses of the Mean Value Theorem. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

Since h is a polynomial, it is continuous on $[-2,2]$ and differentiable on $(-2,2)$. We now want to find all numbers c in $(-2,2)$ such that

$$h'(c) = \frac{h(2)-h(-2)}{2-(-2)} = \frac{h(2)-h(-2)}{4}.$$

$$h(2) = (2)^3 - 3(2) + 2 = 8 - 6 + 2 = 4$$

$$h(-2) = (-2)^3 - 3(-2) + 2 = -8 + 6 + 2 = 0$$

So, we want

$$h'(c) = \frac{4}{4} = 1.$$

$$h'(c) = 3s^2 - 3 = 1$$

$$\Rightarrow \quad 3s^2 = 4$$

$$\Rightarrow \quad s^2 = \frac{4}{3}$$

$$\Rightarrow \quad \boxed{s = \pm \frac{2}{\sqrt{3}}}.$$

2. [15 points] Find the antiderivative G of g that satisfies the given condition.

$$g(u) = 3u^2 + 4u + 4, \quad G(-1) = -1$$

$$G(u) = 3\left(\frac{u^3}{3}\right) + 4\left(\frac{u^2}{2}\right) + 4u + C$$

$$= u^3 + 2u^2 + 4u + C$$

We want to find C where $G(-1) = -1$

$$(-1)^3 + 2(-1)^2 + 4(-1) + C = -1$$

$$-1 + 2 - 4 + C = -1$$

$$-3 + C = -1$$

$$C = 2$$

So, $\boxed{G(u) = u^3 + 2u^2 + 4u + 2.}$

3. [15 points] Find all local maxima/minima of the function $y = 2x^3 - 9x^2 + 12x$. Make sure to state both x and y values.

$$y' = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2)$$

Thus, $y' = 0$ when $x = 1$ & $x = 2$.

Using the chart on the right, we see that $x = 1$ is a local max and $x = 2$ is a local min.

		1		2	
$x-1$	$-$		$+$		$+$
$x-2$	$-$		$-$		$+$
y'	$+$	local max	$-$	local min	$+$

$\underline{x = 1}$

$$y = 2(1)^3 - 9(1)^2 + 12(1)$$

$$= 5$$

$\underline{x = 2}$

$$y = 2(2)^3 - 9(2)^2 + 12(2)$$

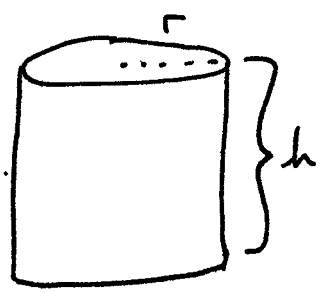
$$= 4$$

$\boxed{\begin{array}{c}\underline{\text{Local max}} \\ (1, 5) \\ \underline{\text{Local min}} \\ (2, 4)\end{array}}$

4. [25 points] A manufacturing executive wants to design a can (= cylinder) which is the cheapest to produce. He decides that this means that the can must have minimal surface area. The can must have a volume of $120cm^3$. Using calculus you must either state the dimensions of the can with minimal surface area or show such a can does not exist.

(Hint: Given a can of radius r and height h, its volume $v = \pi r^2 h$ and its surface area $S = 2\pi r h + 2\pi r^2$)

(You do not need to calculate roots or multiply by the value of π. The answer may be left in the form $\sqrt[3]{300\pi}$ without reducing)



We know $\pi r^2 h = 120 \Rightarrow h = \frac{120}{\pi r^2}$.

Then we can write

$$S(r) = 2\pi r\left(\frac{120}{\pi r^2}\right) + 2\pi r^2$$

$$= \frac{240}{r} + 2\pi r^2$$

Then $$S'(r) = -\frac{240}{r^2} + 4\pi r$$

$$= \frac{4\pi r^3 - 240}{r^3}$$

Since we are only concerned with $r>0$, we want r such that

$S'(r) = 0. \Rightarrow 4\pi r^3 - 240 = 0 \Rightarrow 4\pi r^3 = 240 \Rightarrow r = \sqrt[3]{\frac{60}{\pi}}$.

Since $S'(r) < 0$ for $r < \sqrt[3]{\frac{60}{\pi}}$ and $S'(r) > 0$ for $r > \sqrt[3]{\frac{60}{\pi}}$,

$r = \sqrt[3]{\frac{60}{\pi}}$ is an absolute minimum. Thus, the dimensions

are $r = \sqrt[3]{\frac{60}{\pi}}$ and $h = \frac{120}{\pi\left(\frac{60}{\pi}\right)^{2/3}} = 2\left(\frac{60}{\pi}\right)\left(\frac{60}{\pi}\right)^{-2/3} = 2\sqrt[3]{\frac{60}{\pi}} = 2r.$