MA125-6C Chapter 3 Practice

Name: Key

Exercise 1. *Determine the (a) domain, (b) intercepts, (c) symmetry, (d) asymptotes, (e) intervals of increase or decrease, (f) local maximum and minimum values, and (g) concavity and points of inflection of the following function. Provide a sketch of the function.*

$$y = f(x) = \frac{x}{x^2-9}$$

a) Domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

b) x-int

$y = \frac{(0)}{(0)^2-9} = 0$

$(0,0)$

y-int

$0 = \frac{x}{x^2-9}$

$x = 0$

$(0,0)$

c) $f(-x) = \frac{-x}{(-x)^2-9} = \frac{-x}{x^2-9} = -f(x)$ odd

Symmetry about the origin.

d) Vertical

$\lim_{x\to -3^-} \frac{x}{x^2-9} = -\infty$

$\lim_{x\to -3^+} \frac{x}{x^2-9} = \infty$

$\lim_{x\to 3^-} \frac{x}{x^2-9} = -\infty$

$\lim_{x\to 3^+} \frac{x}{x^2-9} = \infty$

Horizontal

$\lim_{x\to\infty} \frac{x}{x^2-9} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1-\frac{9}{x^2}} = 0$

$\lim_{x\to -\infty} \frac{x}{x^2-9} = \lim_{x\to -\infty} \frac{\frac{1}{x}}{1-\frac{9}{x^2}} = 0$

e) $f'(x) = \frac{(x^2-9)(1) - x(2x)}{(x^2-9)^2}$

$= \frac{-x^2-9}{(x^2-9)^2} = \frac{-(x^2+9)}{(x^2-9)^2}$

	-4	-3	0	3	4
$f'(x)$	-		-		-

Decreasing on $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

f) No local max/min since f' doesn't change signs.

g) $f''(x) = \frac{(x^2-9)^2(-2x) + (x^2+9)2(x^2-9)(2x)}{(x^2-9)^4}$

$= \frac{2x(x^2-9)(-x^2+9+2x^2+18)}{(x^2-9)^4}$

$= \frac{2x(x^2-9)(x^2+27)}{(x^2-9)^4}$

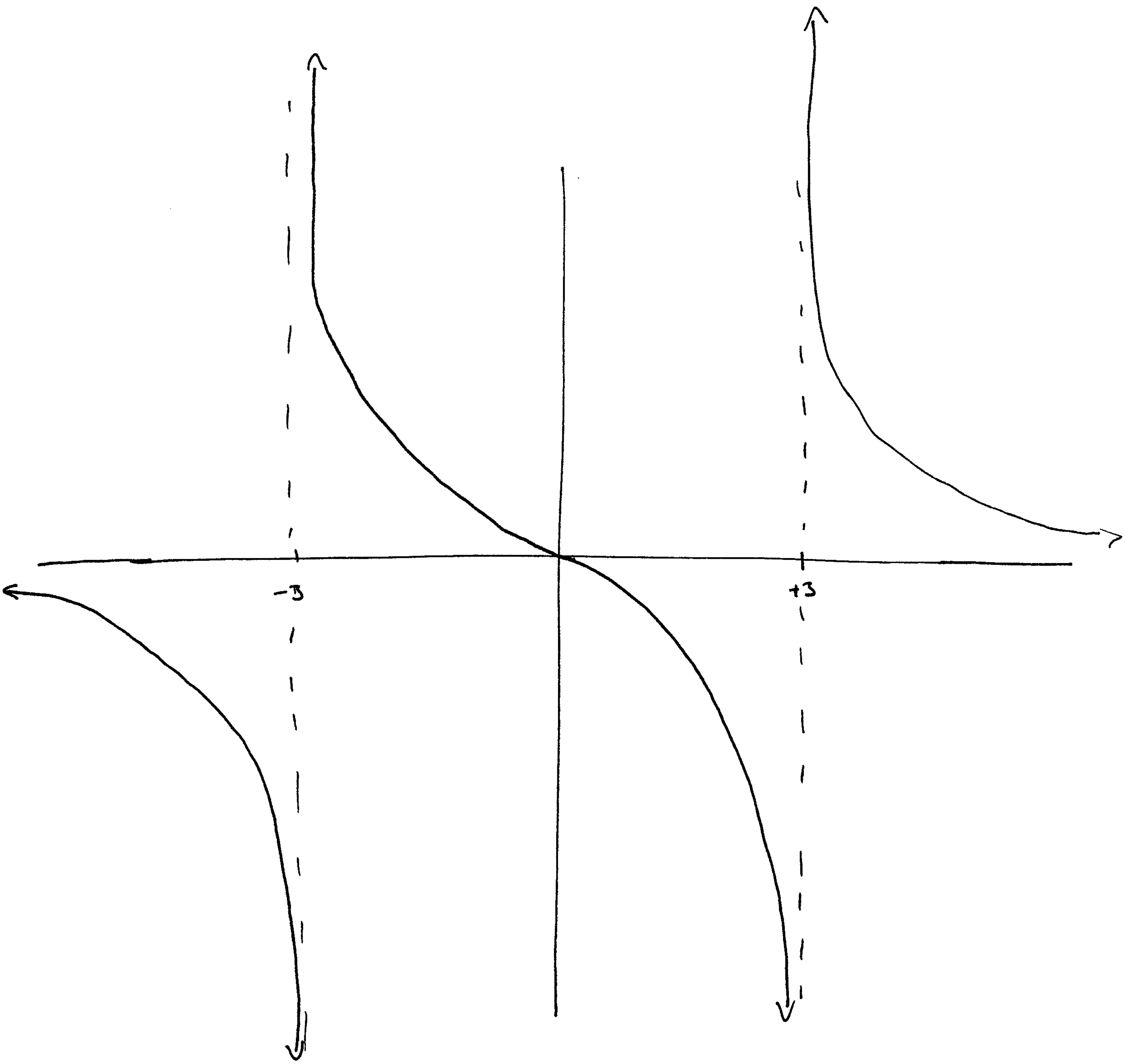
	-4	-3	-1	0	1	3	4
$2x$	-		-		+		+
(x^2-9)	+		-		-		+
$f''(x)$	-		+		-		+

Concave up on $(-3,0)\cup(3,\infty)$

Concave down on $(-\infty,-3)\cup(0,3)$

Inflection pt at $x=0$

No inflection pts at $-3, 3$ due to domain


-3
+3

Exercise 2. *Find two positive numbers whose product is 100 and whose sum is a minimum.*

Let x & y be the numbers. Let S be their sum.

$xy = 100 \Rightarrow y = \frac{100}{x}$

$S = x + y = x + \frac{100}{x}, \quad x > 0$

$S'(x) = 1 - \frac{100}{x^2}$

$= \frac{x^2 - 100}{x^2}$

Critical ~~Pts~~ Pt at

$x = 10$

The values $x = 0$ &

$x = -10$ are not in

the domain.

Since $S'(x) < 0$ for $0 < x < 10$ &

$S'(x) > 0$ for $x > 10$, $S(10)$ is an

absolute minimum. Thus, 10 & 10

are the two ~~num~~ numbers.

Exercise 3. *Show that $2x - 1 - \sin x = 0$ has exactly one real root.*

Let $f(x) = 2x - 1 - \sin(x)$. Then f is continuous everywhere.

Then $f(\pi) = 2\pi - 1 > 0$ & $f(-\pi) = -2\pi - 1 < 0$. The IVT

states that there is at least one root in $(-\pi, \pi)$.

Suppose a & b are both roots of f. Then Rolle's Thm

says there is a c in (a, b) such that $f'(c) = 0$.

But $f'(x) = 2 - \cos(x) \geq 1$. Thus, there is exactly one

real root.